PH 301 Problem Set # 11

Due: Friday, December 5, 2003

1. Transmission line: Griffiths 7.58.
2. Electromagnetic wave: Griffiths 9.9. For part (a) verify explictly that it satisfies Maxwell's equations.
3. Two interfaces: Griffiths 9.34. Hint: To the left, there are the usual incident and transmitted waves. Choose your normalization so that the incident wave has amplitude 1. In the middle, we have *both* a left-moving and a right-moving wave (capturing all the reflections between the surfaces). On the right there is just the transmitted wave. So there are four unknowns: the reflected amplitude, the transmitted amplitude, and the two amplitudes in the middle. You should have two boundary conditions at each of the two boundaries, so you will have four equations for these four unknowns.

   You are welcome to use Mathematica (or Maple or another package you prefer) to solve these equations. As an example, to solve

   $$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned} \tag{1}$$

   for x and y in terms of the constants a, b, c, d, e and f, the Mathematica command is

   ```
   Simplify[Solve[{a x + b y == c, d x + e y == f}, {x, y}]]
   ```

   and don't forget you have to press Shift-Enter! Also, note that you can indicate multiplication with a space or a star: `a x` or `a*x` but NOT `ax`.

   Other Mathematica tips: Enter $\sqrt{-1}$ as capital `I` and the base of the natural logarithm as capital `E`. Built-in functions begin with a capital letter and use square brackets: `Exp[x]`, `Sin[x]`, etc.

   You can get Mathematica to find the magnitude of a complex number, but you have to tell it to assume all unknown variables are real. There's probably a better way to do it, but the following evaluates $|a + ib|$ to $\sqrt{a^2 + b^2}$:

   ```
   ComplexExpand[Abs[a + I b], TargetFunctions -> {Re, Im}]
   ```

   You can also use `Simplify[]` and `FullSimplify[]` to make complicated expressions more manageable.

4. (a) A practical application of the previous problem is the *antireflection coating*, used in camera lenses. In this case, region 1 is the air and region 3 is the glass of the lens. We would like to insert a coating, which is represented by region 2, to maximize the amount of light transmitted into the lens (and eventually to the film). Equivalently, we would like to minimize the amount of light reflected back to the air. Since modern lenses have many air-glass interfaces, antireflection coatings are essential to obtaining a bright image. (In addition, the reflected light can get re-reflected back to the film, leading to an undesirable blurring effect called lens flare.)

   Suppose we fix a frequency of light ω and assume that since region 1 is air, $n_1 = 1$. In terms of ω and $n_3 = n_G$, the index of refraction of glass, find the values of the thickness d and the index of refraction n_2 so that we have perfect transmission, $T = 1$. For a typical optical frequency, find the actual numerical values for these quantities (Griffiths provides the necessary data).

   Hint: Eq. (9.199) is an equation for $1/T$, so we want the right-hand side to be as small as possible in order to make T as big as possible. First choose the optimal d (in terms of ω and n_2), and then find n_2 so that the resulting expression is 1.

   Hint: For the antireflection coating, $1 = n_1 < n_2 < n_3 = n_G$; this fact tells you the sign of the coefficient of the $\sin^2$ term. Use this to figure out what you want $\sin^2$ to do (i.e. should it be 1 or 0), and go from there.

   (b) The closest we can get to the ideal index of refraction with a known material that can be applied as a durable thin layer is to use magnesium difluoride, which has an index of refraction of 1.38. (Because its index of refraction is larger than the ideal value, modern high-quality lenses all use multiple layers of antireflection coatings to further reduce reflections.) Find the fraction of light that is reflected by a magnesium difluoride layer of optimal thickness, and compare that to the fraction reflected in the case of no coating at all.