Math 372A: Seminar in Topology
Differential Algebraic Topology

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1 Introduction

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Brief Description: This will be an introduction to differential manifolds and algebraic topology. Students will be expected to have completed the equivalent of Math 272A (Topology) and Math 283A (Modern Algebra). Topics from the theory of differential manifolds that will be covered include: tangent vectors and differentials, submersions and immersions, transversality, vector bundles, tubular neighborhoods and embeddings in Euclidean space. In algebraic topology we will cover homology and cohomology from the bordism point of view. We hope to get as far as Euler, Chern and Pontrjagin classes and prove the existence of exotic smooth structures on seven dimensional spheres. Note that no previous exposure to homology theory is assumed. On the other hand, since the point of view is different from singular homology, students that have already taken Math 312 (Algebraic Topology) are welcome to take this course. This course is a "must" for students of topology, geometry, noncommutative geometry and operator algebras.

There will be a follow-up course, Math 372B: Seminar in Topology, in Spring 2005. The topic will be Manifold Topology. It will take up where Math 372A ends. The goal is to get to the point where we can understand the background of some of the main problems in topology and geometry: the Baum-Connes Conjectures and the Farrell-Jones Conjectures. These conjectures concern "assembly maps" and, from the topological point of view, the prototypical assembly map is in surgery theory. Surgery theory is one of the main tools in the classification of high dimensional manifolds. We will begin with handlebody decompositions

of smooth manifolds (including the s-cobordism theorem) and Whitehead torsion. We will continue with normal maps and the surgery problem, the algebraic surgery obstruction and the surgery exact sequence. The assembly map lives in the surgery exact sequence.

Here is a list of books you might find useful.

1.1 Texts

1. John W. Milnor, *Topology from the Differentiable Viewpoint*, Princeton University Press, 1997. Available in the bookstore.

2. Matthais Kreck, *Differential Algebraic Topology*, 2002. To be distributed in class.

1.2 History

1. John Dieudonné, *A History of Algebraic and Differential Topology, 1900–1960*, Birkhäuser, 1989.

2. I. M. James (ed.), *History of Topology*, North-Holand, 1999.

1.3 Basics on smooth differential manifolds

1. Dennis Barden and Charles Thomas, *An Introduction to Differential Manifolds*, Imperial College Press, 2003.

2. Glen E. Bredon, *Topology and Geometry*, Springer, 1993. (Chapter II)

3. T. Bröker and K. Jänich, *Introduction to Differential Topology*, Cambridge University Press, 1982.

4. L. Conlon, *Differentiable Manifolds*, Second edition, Birkhäuser, 2001.

5. Victor Guillemin and Alan Pollack, *Differential Topology*, Prentice-Hall, 1974.

6. Morris W. Hirsch, *Differential Topology*, Springer, 1976.

7. Serge Lang, *Differential and Riemannian Manifolds*, Springer, 1995.

8. John M. Lee, *Introduction to Smooth Manifolds*, Springer, 2003.

9. I. Madsen and J. Tornehave, *From Calculus to Cohomology*, Cambridge University Press, 1997.

1.4 More advanced topics on smooth manifolds and topics related to this course

1. Allen Hatcher, *Vector Bundles and K-Theory*, avaiable on line at

   `http://www.math.cornell.edu/~hatcher/VBKT/VBpage.html`

2. Antoni A. Kosinski, *Differential Manifolds*, Academic Press, 1993.

3. John W. Milnor and James D. Stasheff, *Characteristic Classes*, Princeton University Press, 1974.

4. Andrew Ranicki, *Algebraic and Geometric Surgery*, Oxford University Press, 2003.

1.5 Other approaches to algebraic topology

1. Marcelo Aguilar, Samuel Gitler and Carlos Prieto, *Algebraic Topology from a Homotopical Viewpoint*, Springer, 2002.

2. Raoul Bott and Loring W. Tu, *Differential Forms in Algebraic Topology*, Springer, 1982.

3. Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002. Available on line at

   `http://www.math.cornell.edu/~hatcher/AT/ATpage.html`

4. J. Peter May, *A Concise Course in Algebraic Topology*, University of Chicago Press, 1999.

5. Hajime Sato, *Algebraic Topology: An Intuitive Approach*, American Mathematical Society, 1999.